

3.11.1. Duality Problems

A. Consider the formal language $\{ \}$ which has **no connectives**, only sentence letters. What is the **dual** of language $\{ \}$?

B. Construct formal sentences (possibly with truth tables) to do each of the following.

1. Show that the language $\{\leftrightarrow\}$ has a sentence matching the truth table for “ \top ”.
2. On the basis of language duality and your answer to (1), show that the language $\{\oplus\}$ has a sentence matching the truth table of “ \perp ”.
3. Show that the language $\{\leftrightarrow, \perp\}$ has a sentence matching the truth table for “ $\sim P$ ”.
4. On the basis of Tilde Insertion duality and your answer to (3), show that the language $\{\leftrightarrow, \perp\}$ has a sentence matching the truth table for “ $(P \oplus Q)$ ”.
5. On the basis of your answers to (2) and (3), show that the language $\{\leftrightarrow, \oplus\}$ has a sentence matching the truth table for “ $\sim P$ ”.
6. On the basis of language duality and your answer to (3), show that the language $\{\oplus, \top\}$ has a sentence matching the truth table for “ $\sim P$ ”.
7. On the basis of Tilde Insertion duality and your answer to (6), show that the language $\{\oplus, \top\}$ has a sentence matching the truth table for “ $(P \leftrightarrow Q)$ ”.
8. On the basis of language duality and your answer to (4), show that the language $\{\oplus, \top\}$ has a sentence matching the truth table for “ $(P \leftrightarrow Q)$ ”.